Curve Sketching (Book M2A Chapter 6)

Section A

1. Define $f(x) = \dfrac{x^2 - 5}{x - 3}$ for all $x \neq 3$. Denote the graph of $y = f(x)$ by G.

(a) Find the maximum and minimum points and asymptote(s) of G.

(b) Sketch G.

2. Define $f(x) = \dfrac{x^2 - 2x - 12}{x - 4}$ for all $x \neq 4$. Denote the graph of $y = f(x)$ by G.

(a) Find the asymptote(s) of G.

(b) Find the equations of the tangents which are parallel to the straight line $5x - 4y + 1 = 0$.

3. Define $f(x) = \dfrac{A}{x^2 + 2x + 13}$ for all real numbers x, where A is a constant. It is given that the extreme value of $f(x)$ is 4.

(a) Find $f'(x)$.

(b) Somebody claims that there are at least two asymptotes of the graph of $y = f(x)$. Do you agree? Explain your answer.

(c) Find the point(s) of inflexion of the graph of $y = f(x)$.

4. Consider a continuous function $f(x) = \dfrac{2x^2 + 8}{x^2 + 12}$. It is given that

x	$x < -2$	-2	$-2 < x < 0$	0	$0 < x < 2$	2	$x > 2$
$f'(x)$	–	–	–	0	+	+	+
$f''(x)$	–	0	+	+	+	0	–

('+' and '–' denote 'positive value' and 'negative value' respectively.)

(a) Find all the maximum and/or minimum point(s) and point(s) of inflexion.

(b) Find the asymptote(s) of the graph of $y = f(x)$.

(c) Sketch the graph of $y = f(x)$.

5. Consider a rational function $f(x) = \dfrac{(x-1)^3}{(x+1)^2}$. It is given that

x	$x < -5$	-5	$-5 < x < -1$	-1	$-1 < x < 1$	1	$x > 1$
$f'(x)$	+	0	–	undefined	+	0	+
$f''(x)$	–	–	–	undefined	–	0	+

('+' and '–' denote 'positive value' and 'negative value' respectively.)

(a) Find all the maximum and/or minimum point(s) and point(s) of inflexion.

(b) Find the asymptote(s) of the graph of $y = f(x)$.

(c) Sketch the graph of $y = f(x)$.

6. Define $f(x) = \frac{12}{x-3} - \frac{12}{x+3} - 1$ for all $x \neq 3$ and $x \neq -3$. Denote the graph of $y = f(x)$ by G.

(a) Find the x-intercept(s) and y-intercept of G.

(b) (i) Find $f'(x)$ and prove that $f''(x) = \frac{432(x^2+3)}{(x-3)^3(x+3)^3}$.

(ii) Hence find the relative extreme point(s) of G and show that there are no points of inflexion for the graph.

(c) Find the asymptote(s) of G.

(d) Sketch G.

7. Define $f(x) = \frac{-x^2+8x-7}{x+1}$ for all $x \neq -1$. Denote the graph of $y = f(x)$ by G.

(a) Find $f'(x)$ and $f''(x)$.

(b) Find all the relative extreme point(s) of G and the point of inflexion of G.

(c) Find the asymptote(s) of G.

(d) Sketch G.

8. Define $f(x) = \frac{4(x+1)}{(x-1)^3}$ for all $x \neq 1$. Denote the graph of $y = f(x)$ by G.

(a) Find the x–intercept and the y–intercept of G.

(b) (i) Find $f'(x)$ and $f''(x)$.

(ii) Find the relative extreme point(s) and the point(s) of inflexion of G.

(c) Find the asymptote(s) of G.

(d) Sketch G.

9. Let a and b be constants. Define $f(x) = x^3 + ax^2 + bx + 12$ for all real numbers x. Denote the curve $y = f(x)$ by C. It is given that $P(4, -20)$ is a turning point of C.

(a) Find a and b.

(b) Is P a minimum point of C? Explain your answer.

(c) Find the maximum value(s) of $f(x)$.

(d) Find the point(s) of inflexion of C.

(e) Sketch G.

10. Let a and b be constants. Define $f(x) = ax^4 + bx^3 + 2$ for all real numbers x. Denote the curve $y = f(x)$ by C. It is given that $P(3, -79)$ is a turning point of C.

(a) Find a and b.

(b) Is P a minimum point of C? Explain your answer.

(c) Somebody claims that the maximum point of C exists. Do you agree? Explain your answer.

(d) Find the point(s) of inflexion of C.

(e) Sketch G.

