Name: _____ ( ) Class: _____ Date: _____

Lesson Worksheet 8.2B(II)

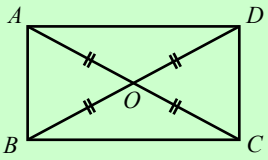
Objective: To learn and use the properties of rectangles.

A rectangle is a quadrilateral with all interior angles equal to 90°.
長方形是一個所有內角的大小都是 90° 的四邊形。
The followings are the properties of rectangles: 以下是長方形的性質：

(1) All properties of a parallelogram. 平行四邊形的所有性質。

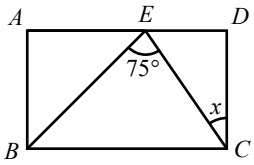
(2) The diagonals are equal. 對角線相等。
i.e. $AC = BD$

(3) The diagonals bisect each other into four equal line segments.
對角線互相平分為四個相等的線段。
i.e. $OA = OB = OC = OD$



[Ref.: *property of rectangle*] 〔簡記：長方形性質〕

1. In the figure, $ABCD$ is a rectangle. E is a point on AD such that $AB = AE$ and $\angle BEC = 75°$. Find the value of x.



$\angle BAD = \angle CDA = 90°$ (*definition of rectangle*)

In $\triangle ABE$,

$\because$ $AB = AE$ (*given*)

$\therefore$ $\angle ABE = \angle AEB$ (*base ∠s, isos. △*)

$\angle ABE + \angle AEB + 90° = 180°$ (*∠ sum of △*)

$2\angle AEB =$ 90°

$\angle AEB =$ 45°

$\angle CED + 75° +$ 45° $= 180°$ (*adj. ∠s on st. line*)

$\angle CED = 60°$

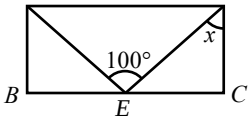
In $\triangle CDE$,

$x +$ 90° $+$ 60° $=$ 180° (*∠ sum of △*)

$x =$ 30°

Demonstration

In the figure, $ABCD$ is a rectangle. E is a point on BC such that $AE = DE$ and $\angle AED = 100°$. Find the value of x.



Solution

In $\triangle ADE$,

$\because$ $AE = DE$ (*given*)

$\therefore$ $\angle EAD = \angle EDA$ (*base ∠s, isos. △*)

$\angle EAD + \angle EDA + 100° = 180°$ (*∠ sum of △*)

$2\angle EDA = 80°$

$\angle EDA = 40°$

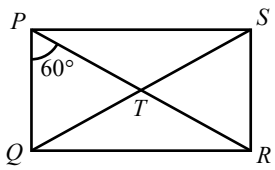
$\angle ADC = 90°$ (*definition of rectangle*)

$x = \angle ADC - \angle EDA$

$= 90° - 40°$

$= 50°$

2. In the figure, $PQRS$ is a rectangle. PR and QS intersect at T. $\angle QPT = 60°$. Find $\angle RTS$.



In $\triangle PQT$,

$\because$ $PT = QT$ (*property of rectangle*)

$\therefore$ $\angle PQT = \angle QPT =$ 60° (*base ∠s, isos. △*)

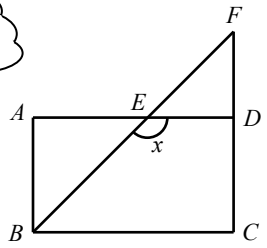
$\angle PTQ + \angle PQT + \angle QPT = 180°$ (*∠ sum of △*)

$\angle PTQ + 60° + 60° = 180°$

$\angle PTQ = 60°$

$\angle RTS = \angle PTQ =$ 60° (*vert. opp. ∠s*)

3. In the figure, $ABCD$ is a rectangle. AED, BEF and CDF are straight lines. $BC = CF$. Find the value of x.



$\angle BCF = 90°$ (*definition of rectangle*)

In $\triangle BCF$,

$\because$ $BC = CF$ (*given*)

$\therefore$ $\angle CBF =$ $\angle CFB$ (*base* $\angle$*s, isos.* $\triangle$)

$\angle CBF + \angle CFB + \angle BCF = 180°$ ($\angle$ *sum of* $\triangle$)

$2\angle CBF + 90° = 180°$

$\angle CBF = 45°$

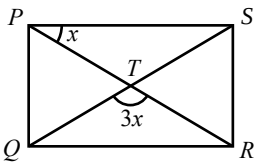
$\angle DEF = \angle CBF =$ $45°$ (*corr.* $\angle$*s,* AD // BC)

$\angle BED + \angle DEF = 180°$ (*adj.* $\angle$*s on st. line*)

$x + 45° = 180°$

$x = 135°$

4. In the figure, $PQRS$ is a rectangle. PR and QS intersect at T. Find the value of x.



$\angle PTS =$ $3x$ (*vert. opp.* $\angle$*s*)

In $\triangle PTS$,

$\because$ $PT = ST$ (*property of rectangle*)

$\therefore$ $\angle PST = \angle SPT =$ x (*base* $\angle$*s, isos.* $\triangle$)

$\angle PTS + \angle PST + \angle SPT = 180°$ ($\angle$ *sum of* $\triangle$)

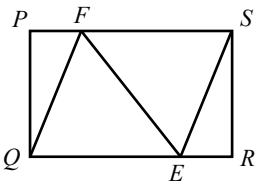
$3x + x + x = 180°$

$5x = 180°$

$x = 36°$

Challenging Question(Optional)

5. In the figure, $PQRS$ is a rectangle. E and F are points on QR and PS respectively such that $EQ = FQ$ and $PF = RE$. Prove that $EQFS$ is a rhombus.



$PF =$ RE	*given*
$PQ =$ RS	*property of rectangle*
$\angle QPF = \angle SRE =$ $90°$	*definition of rectangle*
$\therefore$ $\triangle PQF \cong \triangle RSE$	SAS
$FQ =$ ES	*corr. sides,* $\cong\triangle$*s*
$PS =$ QR	*property of rectangle*
$FS = PS - PF$	
$=$ QR $-$ RE	
$=$ EQ	
$\because$ $EQ = FQ$	*given*
FS $=$ ES $= EQ = FQ$	
$\therefore$ $EQFS$ is a rhombus.	*by definition*