Name: _____ ( ) Class: _____ Date: _____

Lesson Worksheet 13.4

Objective: To use the properties of vector product to find the vector product of two vectors.

For any two vectors $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ in three-dimensional space,

$\mathbf{a} \times \mathbf{b} = (a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$

Alternatively,

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\mathbf{k}$$

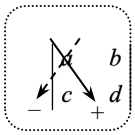
1. Find the following vector products.

(a) $(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - \mathbf{k})$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 3 \\ \square & \square & \square \end{vmatrix}$$

◄ 1st vector

◄ 2nd vector



$$= \begin{vmatrix} 2 & 3 \\ \square & \square \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & 3 \\ \square & \square \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & 2 \\ \square & \square \end{vmatrix}\mathbf{k}$$

$=$

(b) $(3\mathbf{i} + \mathbf{j} - 2\mathbf{k}) \times (\mathbf{i} - 4\mathbf{k})$

For any vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ in three-dimensional space, and scalar λ, the following are true.

1. $\mathbf{a} \times \mathbf{a} = \mathbf{0}$

2. $\mathbf{a} \times \mathbf{b} = -(\mathbf{b} \times \mathbf{a})$

3. $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$

4. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$

5. $(\lambda\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (\lambda\mathbf{b}) = \lambda(\mathbf{a} \times \mathbf{b})$

6. $|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2$

2. It is given that $\mathbf{a} = 2\mathbf{i} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$. Find the following vector products.

(a) $\mathbf{a} \times \mathbf{b}$

(b) $(2\mathbf{b}) \times \mathbf{a}$

(a) $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \square & \square & \square \\ \square & \square & \square \end{vmatrix}$

$=$

(b) $(2\mathbf{b}) \times \mathbf{a} = (\quad)(\mathbf{b} \times \mathbf{a})$ ◄ Use property 5.

$= (\quad)[(\quad)(\mathbf{a} \times \mathbf{b})]$ ◄ Use property 2.

$=$

3. It is given that $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = -\mathbf{i} + 2\mathbf{j} + \mathbf{k}$. Find the following vector products.

(a) $\mathbf{a} \times \mathbf{b}$ **(b)** $(2\mathbf{b}) \times (3\mathbf{a})$ →Exercise 13.4: 9

4. It is given that $\mathbf{a} = -3\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

(a) Find $\mathbf{a} \times \mathbf{b}$.

(b) If $\mathbf{c} = \mathbf{a} + \mathbf{b}$ and $\mathbf{d} = \mathbf{a} + 4\mathbf{b}$, find $\mathbf{c} \times \mathbf{d}$.

$\mathbf{c} \times \mathbf{d}$

$= (\quad) \times (\quad)$

$\mathbf{a} \times \mathbf{a} = ?$
$\mathbf{b} \times \mathbf{b} = ?$

5. It is given that $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 6\mathbf{j} + 4\mathbf{k}$. →Exercise 13.4: 18, 19

(a) Find $\mathbf{a} \times \mathbf{b}$.

(b) If $\mathbf{c} = \mathbf{a} - 2\mathbf{b}$ and $\mathbf{d} = 3\mathbf{a} - \mathbf{b}$, find $\mathbf{c} \times \mathbf{d}$.

Try More

6. If the angle between two vectors $\mathbf{a}$ and $\mathbf{b}$ is 60°, $|\mathbf{a}| = 4$ and $|\mathbf{b}| = 5$, find $|\mathbf{a} \times \mathbf{b}|$.